

## Soil vibration by high speed trains: an evaluation method

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### ABSTRACT

No suitable handy tool is available to predict train induced vibration on environmental impact assessment. A simple prediction model is proposed which has been calibrated for high speed trains. The model input data are train characteristics, train speed and track properties; model output data are soil time averaged velocity and velocity level. The proposed model has been calibrated by means of a measurement campaign led along an Italian high speed rail line. Vibration have been measured during ETR500 passage. Model results are obtained in terms of maximum soil r.m.s. velocity and vibration level. The model can be validated by comparing high speed train measured vibration levels with the predicted ones.

### INTRODUCTION

Environmental impact assessments require the prediction of vibration induced by moving loads like trains or trucks [1] [2]. Although complicated simulation procedures, which require long time calculation and a huge amount of input data are known, no easy-usage method is available to predict train induced vibrations [3]. On environmental impact assessments vibrations are very often predicted without a mathematical method but simply describing possible macroscopic effect induced by vibration [4]. In this paper a useful simple method for train-induced vibration prediction is proposed. The method has been introduced for high speed trains. The model input data are train velocity, train mass, rail geometry, soil characteristics and prediction point-rail distance. Model results are furnished in terms of maximum soil r.m.s. velocity and vibration level. Model formulation is attained by adopting sustainable hypothesis. The model has been calibrated by means of measurement results led along an Italian railway. Since calibrating constant is set, the model maximum error, in terms of vibration level, is lower than 0.75dB. Model is actually under comparison to a lot of experimental data retrieved from many measurement campaign carried out along the most important high-speed European railways.

#### 1. VIBRATION SOURCE MODEL: THEORETICAL FORMULATION

In order to estimate soil r.m.s. velocity at a prediction point both vibration source and propagation phenomena must be modeled.

Energy transferred from vibration source (train and embankment) to soil is an instantaneous quantity which is governed by complex mechanism the behavior of which is difficultly identifiable [5] [6]. Thus, a simplifying hypothesis is introduced: a constant portion of the time averaged power transferred by the traveling train to the ballast-embankment system is then retransferred to the surrounding soil. Such a hypothesis is verified if ballast-

embankment system and junction elements are uniform all over the railway; thus the following statement may be written:

$$W_0 = K \cdot W_T. \quad (1)$$

The constant  $K$  must be set by calibrating the model. The symbols are described in Table 2.

Time averaged power  $W_T$  transferred by train to ballast-embankment system depends on train velocity, train mass, train length and rail geometry. In order to determine  $W_T$ , train specific mass is introduced which is defined as follows:

$$m = \frac{M}{T}. \quad (2)$$

Time averaged energy transferred by train per unit of length is given by:

$$e = m \cdot g \cdot s, \quad (3)$$

where  $s$  is the maximum vertical rail displacement admitted for a train passage [7] [8] [9]. Maximum vertical displacement is chosen in order to adopt a conservative assumption. Power is found assuming that energy is transferred to the ballast-embankment system by means of the sleepers; furthermore energy expressed by Eq. (3) is released during the time the train takes to go from a sleeper to the next one. Thus the power associated to Eq. (3) is given by:

$$w_T = m \cdot g \cdot s \cdot \frac{v_T}{i}. \quad (4)$$

Power transferred to ballast-embankment system by a  $dx$  portion of train is:

$$dW_T = m \cdot g \cdot s \cdot \frac{v_T}{i} \cdot dx. \quad (5)$$

According to Eq. (1), the time averaged power retransferred from the ballast-embankment system to the soil by a  $dx$  portion of train is given:

$$dW_0 = m \cdot g \cdot s \cdot \frac{v_T}{i} \cdot K \cdot dx. \quad (6)$$

## 2. PROPAGATION MODEL

Vibration waves propagation has been modeled adopting the following hypothesis:

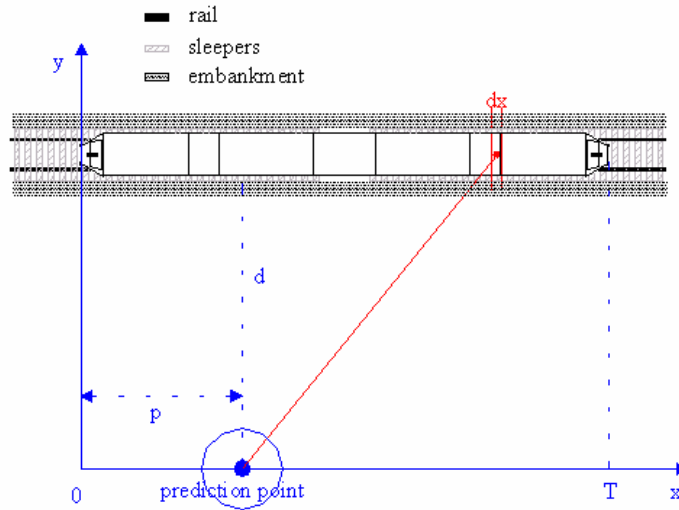
- A. Embankment is considered to be a continuous emitting source the length of which corresponds to train length [10] [11] [12] [13]. Source time averaged power per unit of length is expressed by Eq. (6).
- B. Vibrational energy is transported only on soil surface by means of Rayleigh waves since their amplitude exponentially decreases along a vertical direction, perpendicular to soil surface [14]. Primary, Secondary and Love waves are not kept into account [15].
- C. Each source elementary portion is a point source which is characterized by a superficial omnidirectional vibrational energy emission [16].

When a not dissipative media is considered, vibrational energy transported by Rayleigh waves through soil surface decreases proportionally to  $1/r$ . According to hypothesis A, B, C, time averaged intensity at a generical point  $P$  may be found by calculating power which passes through a  $P$  centered unitary diameter circle (see Fig.1):

$$dJ_r = \frac{dW_r}{2 \cdot \pi \cdot r}, \quad (7)$$

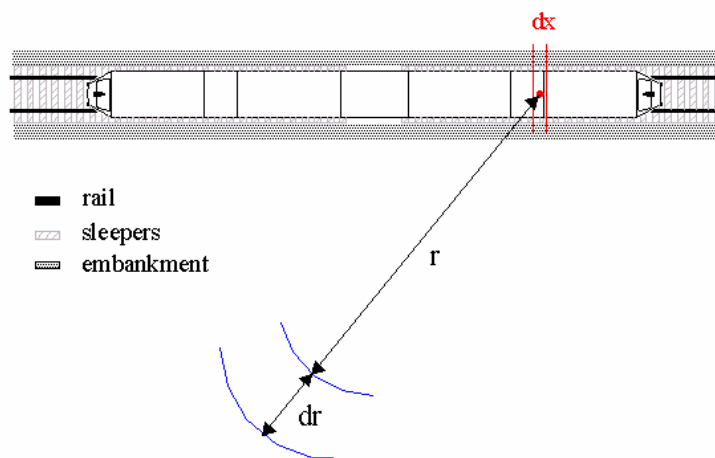
where  $dW_r$  represents the total power produced by an elementary source which is available at  $r$  distance from the elementary source itself. Intensity at point P, due to the entire train, is given by:

$$J_r = \int_0^T \frac{dW_r}{2 \cdot \pi \cdot r}. \quad (8)$$



**Figure 1. Model reference scheme**

$dW_r$  may be determined as follows (see Fig.2): let's consider the power which is dissipated inside an elementary ring area the center of which is the source; the ring inner radius is  $r$ , the outer one is  $r+dr$ . Dissipated power may be calculated as the difference between the ring entering and exiting power.



**Figure 2. Energy dissipation reference scheme**

Furthermore, dissipated power is proportional to power which enters into the ring, ring thickness and soil properties which are characterized by a dissipation constant [17].

$$-d(dW_r) = -(dW_{r+dr} - dW_r) = dW_r \cdot \alpha \cdot dr. \quad (9)$$

Integrating Eq.(9), the following equation is obtained:

$$dW_r = dW_0 \cdot e^{-\alpha \cdot r}. \quad (10)$$

Combining Eq. (6) and Eq. (9), we have:

$$dW_r = m \cdot g \cdot s \cdot \frac{v_T}{i} \cdot K \cdot e^{-\alpha \cdot r} \cdot dx. \quad (11)$$

According to Eq. (11), Eq. (8) becomes:

$$J_r = \int_0^T \frac{m \cdot g \cdot s \cdot v_T \cdot K \cdot e^{-\alpha \cdot r}}{2 \cdot \pi \cdot i \cdot r} \cdot dx. \quad (12)$$

According to the reference scheme (see Fig.1), maximum value of intensity  $J_r$  is attained when  $p=T/2$ , i.e., train position is symmetrical with respect to a train perpendicular line which crosses point P. Thus,  $J_{max}$  is given by:

$$J_{max} = \int_{-T/2}^{T/2} \frac{m \cdot g \cdot s \cdot v_T \cdot K \cdot e^{-\alpha \cdot \sqrt{x^2 + d^2}}}{2 \cdot \pi \cdot i \cdot \sqrt{x^2 + d^2}} dx. \quad (13)$$

Since  $J_{max}$  is the time averaged maximum power which crosses a  $P$  centered unitary diameter circle, the maximum time averaged vibration energy density at point  $P$  is:

$$D_{max} = \frac{J_{max}}{c_R}. \quad (14)$$

Absolute value of particles r.m.s. velocity is attained by combining Eq. (14) by the following relation [18] [19]:

$$D_{max} = \rho_s \cdot u^2. \quad (15)$$

Thus:

$$u = \sqrt{\frac{J_{max}}{\rho_s \cdot c_R}}. \quad (16)$$

Absolute vibration level is then given by the following relation:

$$L = 10 \cdot \log \left( \frac{u}{u_{ref}} \right)^2; \quad u_{ref} = 10^{-8} m/s. \quad (17)$$

Rayleigh waves propagation velocity  $c_R$  is defined as written below [20]:

$$c_R = C \cdot \sqrt{\frac{G}{\rho}} \quad (18)$$

$C$  may be found by solving the following equation [20]:

$$C^6 - 8 \cdot C^4 + 8 \cdot \left(3 - \frac{1-2\nu}{1-\nu}\right) \cdot C^2 - 16 \cdot \left(1 - \frac{1-2\nu}{2 \cdot (1-\nu)}\right) = 0 \quad (19)$$

In Eq. (18), soil torsional elasticity module  $G$  is defined as shown by eq. (20):

$$G = \frac{E}{2(1+\nu)} \quad (20)$$

**Table 1. Table of Symbols**

Symbol	Units	Description
$\alpha$	$\text{m}^{-1}$	soil dissipation constant
$C$	adimensional	Rayleigh waves propagation velocity constant
$c_R$	$\text{m} \cdot \text{s}^{-1}$	Rayleigh waves propagation velocity
$D_{max}$	$\text{J} \cdot \text{m}^{-2}$	maximum averaged vibration energy density
$d$	$\text{m}$	minimum distance between rail and prediction point
$E$	$\text{Pa}$	soil Young module
$e$	$\text{J} \cdot \text{m}^{-1}$	energy transferred by train to ballast-embankment system per unit of length
$G$	$\text{Pa}$	Soil torsional elasticity module
$g$	$\text{m} \cdot \text{s}^{-2}$	gravity acceleration
$i$	$\text{m}$	distance between two consecutive sleepers
$J_{max}$	$\text{W} \cdot \text{m}^{-1}$	average vibration intensity transferred when train position is symmetrical with respect to a train perpendicular line which crosses prediction point
$J_r$	$\text{W} \cdot \text{m}^{-1}$	average vibration intensity transferred by train to surrounding soil at $r$ distance
$K$	adimensional	model calibrating constant
$L$	$\text{dB}$	absolute vibration level
$M$	$\text{Kg}$	train total mass
$m$	$\text{Kg} \cdot \text{m}^{-1}$	train specific mass
$p$	$\text{M}$	distance between $x=0$ and prediction point (see Fig.2)
$r$	$\text{M}$	distance between a soil surface point and a source one
$\rho$	$\text{Kg} \cdot \text{m}^{-3}$	soil density
$\rho_s$	$\text{Kg} \cdot \text{m}^{-2}$	soil superficial density
$s$	$\text{M}$	maximum vertical rail displacement
$T$	$\text{M}$	train length
$u$	$\text{m} \cdot \text{s}^{-1}$	particles r.m.s. velocity
$u_{ref}$	$\text{m} \cdot \text{s}^{-1}$	reference particles velocity
$\nu$	Adimensional	soil Poisson's ratio
$v_T$	$\text{m} \cdot \text{s}^{-1}$	train speed
$W_0$	$\text{W}$	power transferred by train to surrounding soil
$W_r$	$\text{W}$	power transferred by train to soil at $r$ distance
$W_{r+dr}$	$\text{W}$	power transferred by train to soil at $r+dr$ distance
$W_T$	$\text{W}$	power transferred by train to ballast-embankment
$w_T$	$\text{W} \cdot \text{m}^{-1}$	power transferred by train to ballast-embankment per unit of length
$x$	$\text{M}$	portion of train

### 3. MODEL CALIBRATION

The proposed model has been calibrated by means of measurements campaign. Calibration is needed in order to determine the value of constant  $K$  (see Eq. (1) ).

#### 3.1 Measurement Campaign

Measurement points have been placed along a straight part of an high speed rail line sited close to Terontola Station in Italy (Rome-Florence stage); no bridge and no bend are located near the measurement points. Vibrations have been measured at 5, 10, 20, 40 meters from the rail center line. Soil particles velocity components has been measured. Measurements have been carried out by means of geophones: models GS-32CT for x and y components, GS-30CT for z component [21]. Geophones signals have been acquired and processed by means of OROS acquisition board connected to a custom DASY-LAB based code for vibration signal processing [22]. Signal processing allowed to calculate particles r.m.s. velocities from the train-induced instantaneous velocities time history. Particles r.m.s. velocity value has been calculated by the code according to the following relation:

$$\begin{aligned}
 u_x &= \frac{1}{N} \sum_{j=0}^N u_{x,j}; \quad \text{with} \quad u_{x,j} = \sqrt{\int_j^{j+1} v_{x,j}^2(t) \cdot dt} \\
 u_y &= \frac{1}{N} \sum_{j=0}^N u_{y,j}; \quad \text{with} \quad u_{y,j} = \sqrt{\int_j^{j+1} v_{y,j}^2(t) \cdot dt} \\
 u_z &= \frac{1}{N} \sum_{j=0}^N u_{z,j}; \quad \text{with} \quad u_{z,j} = \sqrt{\int_j^{j+1} v_{z,j}^2(t) \cdot dt}
 \end{aligned} \tag{21}$$

The data processing system furnishes also a vibration level calculated on 1 second time interval. For the  $j$ -th 1 second time interval, vibration level is defined as follows:

$$L_j = 10 \cdot \log \left( \frac{u_j}{u_{ref}} \right)^2 = 20 \cdot \log \left( \frac{\sqrt{u_{x,j}^2 + u_{y,j}^2 + u_{z,j}^2}}{u_{ref}} \right); \quad u_{ref} = 10^{-8} \text{ m/s} \tag{22}$$

The time  $N$  is the total measurement integration time which is different for each train passage. Within such an interval each 1 second vibration level  $L_j$  is higher than 10dB:

$$L_j > 10\text{dB} \tag{23}$$

Condition (23) means that outside the  $N$  time interval train passage produces vibrations levels lower than 10dB. Vibration have been measured during ETR500 passage. ETR500 is an Italian high speed train the characteristics of which are:  $M=620 \cdot 10^3 \text{Kg}$ ,  $T=328\text{m}$ . The soil which surrounds the measurement site is composed by compressed high density sands the characteristics of which are  $E=90 \cdot 10^6 \text{Pa}$  and  $\nu=0.2$  [23].

#### 3.2 Calibration

The model has been calibrated by determining the value of  $K$  (see Eq.(1)) which equalizes model result (level furnished by Eq. (17)) to the corresponding measured values. Calibration is carried out setting maximum rail vertical displacement  $s=1 \cdot 10^{-2} \text{m}$  which is the maximum admitted value [7]. For each different train speed and rail-point distance, the ‘‘equalizing’’  $K$  assumes a different value. Each  $K$  is, however, very close to its average value (see Table 2). Thus the model may be calibrated setting:

$$K = 2 \cdot 10^{-6} \quad (24)$$

According to Table 2, absolute maximum difference with respect to average is  $3.0 \cdot 10^{-7}$ , and  $K$  standard deviation is  $1.8 \cdot 10^{-7}$ .

**Table 2. Values of  $K$  which equalizes model result to measurement ones**

Rail-point distance [m]	Train velocity [km/h]		
	100	150	150
	K value	K value	K value
5	$1.9 \cdot 10^{-6}$	$1.8 \cdot 10^{-6}$	$2.0 \cdot 10^{-6}$
10	$2.2 \cdot 10^{-6}$	$2.0 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$
20	$1.7 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$1.7 \cdot 10^{-6}$
40	$1.9 \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$	$2.1 \cdot 10^{-6}$
Average $\underline{K} = 1.983 \cdot 10^{-6} \approx 2 \cdot 10^{-6}$ $\sigma_k = 1.8 \cdot 10^{-7}$ $ K - \underline{K} _{\max} = 3.0 \cdot 10^{-7}$			

Calibrating the model according to (24), maximum error, in terms of predicted vibration level, is lower than 0.75dB. According to the previous observation,  $K$  is supposed invariant for any high speed rail lines; it may be admitted because track ballast and embankment are characterized by the same construction criteria for any high speed railways [24].

## CONCLUSIONS

A simple method to predict soil vibration levels induced by high speed train is proposed. In order to estimate soil r.m.s. velocity at a prediction point both vibration source and propagation phenomena has been modeled. The proposed model requires short calculation time and a small amount of input data.

The model has been calibrated by means of a measurement campaign led along an Italian high speed railway during ETR 500 passage. For each different train speed and rail-point distance, the calibration constant  $K$  assumes a different values. Each  $K$  is however very close to its average value ( $2 \cdot 10^{-6}$ ); calibration is attained by setting  $K$  equal to the average value. Maximum error, in terms of vibration level, is lower than 0.75dB when the proposed model is applied to the same railway which has been employed for calibration. The model is actually going to be compared to a great amount of vibration level data retrieved from the reports of experimental investigations led along the most important European high speed railways.

## REFERENCES

1. U.S. Department of Transportation. Transit Noise and Vibration Impact Assessment. Federal Transit Administration, Report DOT-T-95-16, 1995.
2. A. Crone, T. Astrup, P. Finne. Prediction of Vibrations and Structure-Borne Noise from Railways. InterNoise 99, Florida, USA, 1999.
3. T. Ekevid, M.X.D. Li, N. Wiberg. Adaptive Finite Element Analysis of Wave Propagation Under Moving Loads Induced by High Speed Trains. ECCOMAS 2000, Barcelona, 2000.
4. H. Takemiya. Simulation for Vibration Prediction and Mitigation of Track-Ground due to Highspeed Trains - Case of X-2000 in Sweden -. Informal Workshop at Royal Institute of Technology, Sweden, 2001.
5. F.E. Richert, J.R. Hall. Vibrations of Soils and Foundations. Prentice-Hall Inc., Englewood Cliffs, NJ, 1970.

6. C.G. Lai, A. Callerio, E. Faccioli, A. Martino. Mathematical Modeling of Railway-Induced Ground Vibrations. WAVE 2000, Bochum, Germany, 2000.
7. M.E. Heelis, A.C. Collop, A.R. Dawson, D.N. Chapman, V. Krylov. Predicting and Measuring Vertical Track Displacements on Soft Subgrades. Railway Engineering 99, London, 1999.
8. L. Fryba. Vibration of Solids and Structures under Moving Loads. Telford, London, 1999.
9. J.P. Fortin. Dynamic Track Deformation. French Railway Review. Vol. 1, 1983.
10. H. Takemiya. Prediction of Ground Vibration Induced by High-Speed Train Operation. 18<sup>th</sup> Sino-Japan Technology Seminar, Taipei, Taiwan, 1997.
11. D. Le Houdec. Modelling and Analysis of Ground Vibration Problems: a Review. Civil and Structural Engineering Computing, Chapter 19, 2001.
12. H.E.M. Hunt. Measurement and Modelling of Traffic Induced Ground Vibration. Ph.D. Thesis, Cambridge University, England, 1988.
13. T.G. Gutowski, L.E. Wittig, C.L. Dym. Some Aspects of the Ground Vibration Problem. Noise Control Engineering, vol. 10:3, 1978.
14. H. Hung, Y. Yang. A Review of Researches on Ground-Borne Vibrations with Emphasis on Those Induced by Trains. Proc. Natl. Sci. Council., Vol. 25, No.1, 2001.
15. V.V. Krylov, A.R. Dawson, M.E. Heelis, A.C. Collop. Rail Movement and Ground Waves Caused by High-Speed Trains Approaching Track-Soil Critical Velocities. Proc. Instn. Mech. Engrs., Vol. 14, Part F, 2000.
16. L.L. Beranek. Noise and Vibration Control. edited by L.L. Beranek, 1988.
17. G. DeGrande. Free Field Vibration Measurements During the Passage of a Thalys High Speed Train. Katholieke Universiteit Leuven, Report BWM-2000-06, 2002.
18. G. Moncada Lo Giudice, S. Santoboni. Acustica. Masson Editoriale, 1995.
19. R. Spagnolo. Manuale di Acustica. UTET Libreria, Torino, 2001.
20. S.P. Timoshenko. Theory of Elasticity. Mc Graw Hill Inc., USA, 1970.
21. Geo Space. Geo Space Geophones GS-30CT & GS-32CT. [www.geospacehelp.com/gs30ct.htm](http://www.geospacehelp.com/gs30ct.htm), 2002.
22. DASYTec. DASYLab User Guide. [www.dasytec.com](http://www.dasytec.com), 2001.
23. A.V. Damiani, G. Minelli, G. Pialli. L'Unita' Falterona - Trasimeno nell'area compresa fra la Val di Chiana e la Valle Tiberina: Sezione Terontola - Abbazia di Cassiano. Geological Chart, Studi Geologici Camerti, 2000.
24. M.E. Heelis, A.C. Collop, A.R. Dawson, D.N. Chapman, V. Krylov. Resilient Modulus of Soft Soil Beneath High Speed Rail Lines. Transportation Research Board 99, Washington D.C., 1999.